BIRZEIT UNIVERSITY
MATHEMATICS DEPARTMENT
MATH 235
FINAL EXAM//Fall 2011

STUDENT NAME ANAN ABU SAADA

STUDENT NUMBER KEY

INSTRUCTOR NAME

SECTION

100

Question #	score
1	
2	
3	
4	
TOTAL	

Question # 1 (51%) Circle the correct answer

1. Find the slope of the line passes through $(-5,0),(0,5)$
   - a. 1
   - b. -1
   - c. 0
   - d. The line has no slope

$m = \frac{5-0}{0-(-5)} = \frac{5}{5} = 1$

2. The slope of the line tangent to the curve $2x^2 - y^2 = 6x$ at $(1,2)$ is
   - a. 2
   - b. -2
   - c. $\frac{1}{2}$
   - d. $\frac{-1}{2}$

المشتقة $4x - 2yy' = 6$

$\Rightarrow 2yy' = 4x - 6$

$\Rightarrow y' = \frac{4x-6}{2y} = \frac{4(1)-6}{2(2)} = \frac{-2}{4} = -\frac{1}{2}$

3. If $y = (x^3+5)^6$ then $\frac{dy}{dx} =$
   - a. $12x(x^3+5)^5$
   - b. $18x^2(x^3+5)^7$
   - c. $(x^3+5)^5$
   - d. None of the above

$\frac{dy}{dx} = 6(x^3+5)^5 3x^2 = 18x^2(x^3+5)^5$

4. If $\frac{1}{4}\ln(x+4) = 1$ then x =
   - a. e^4+4
   - b. e^4-4
   - c. $e^{\frac{1}{4}}+4$
   - d. None of the above

$\Rightarrow \ln(x+4)^{\frac{1}{4}} = 1 \Rightarrow e^{\ln(x+4)^{\frac{1}{4}}} = e^1$

$\Rightarrow \left((x+4)^{\frac{1}{4}}\right)^4 = (e^1)^4$

$\Rightarrow x+4 = e^4$

$\Rightarrow x = e^4 - 4$

5. If $f(x) = \sqrt[4]{2x+6}$ then $f'(5) =$
   - a. 2
   - b. $\frac{1}{32}$
   - c. $\frac{1}{16}$
   - d. None of the above

$(2x+6)^{\frac{1}{4}}$

$f'(x) = \frac{1}{4}(2x+6)^{-3/4} 2 = \frac{1}{2}(2x+6)^{-3/4} = \frac{1}{2\sqrt[4]{(16)^3}} = \frac{1}{2(8)} = \frac{1}{16}$

6. If $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} a & b \end{bmatrix}$ then $AB =$
   - a. $\begin{bmatrix} a & 2a \\ b & 2b \end{bmatrix}$
   - b. $\begin{bmatrix} a & b \\ 2a & 2b \end{bmatrix}$
   - c. $[a+2b]$
   - d. None of the above

$\begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2\times1} \begin{bmatrix} a & b \end{bmatrix}_{1\times2} = \begin{bmatrix} a & b \\ 2a & 2b \end{bmatrix}_{2\times2}$

لا أريد أن أقول لكم وداعاً، بل إلى اللقاء !!

a. 1000
b. 1800
c. 2000
d. None of the above

$MR = 2000 - 10x$
$= 2000 - 10(100)$
$= 1000$

8. If the demand curve is $p = 40 - 0.1q$ and the supply curve is $p = 0.2q + 10$, the equilibrium point is
a. (100,30)
b. (30,100)
c. (30,30)
d. (100,100)

Demand = supply
$\Rightarrow 40 - 0.1q = 0.2q + 10$
$\Rightarrow 30 = 0.3q \Rightarrow q = \frac{30}{0.3} = 100$
$P(100) = 40 - 0.1(100)$
$= 40 - 10 = 30$
$(100, 30)$

9. If $y = \sqrt{e^x}$ then $\frac{dy}{dx} =$
a. $\frac{\sqrt{e^x}}{2\sqrt{x}}$
b. $\frac{2}{\sqrt{e^x}}$
c. $\frac{1}{2}\sqrt{e^x}$
d. None of the above

$y = (e^x)^{\frac{1}{2}}$
$y' = \frac{1}{2}(e^x)^{-\frac{1}{2}} e^x = \frac{e^x}{2\sqrt{e^x}} = \frac{\sqrt{e^x}\cancel{\sqrt{e^x}}}{2\cancel{\sqrt{e^x}}} = \frac{\sqrt{e^x}}{2}$

10. The vertex of the parabola $f(x) = -x^2 - 2x$ is
a. (−1,−3)
b. (−1,1)
c. (1,−1)
d. None of the above

$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \Rightarrow \frac{2}{2(-1)} = -1$
$f(-1) = -1 + 2 = 1$
$(-1, 1)$

11. If log (5x) + log (x+1) = 1, then x =
a. $\frac{1}{4}$
b. 2
c. 1
d. $\frac{-3}{4}$

$\log 5x(x+1) = 1$
$\Rightarrow \log_{10}(5x^2 + 5x) = 1 \Rightarrow 10 = 5x^2 + 5x \Rightarrow 5x^2 + 5x - 10 = 0$
$x^2 + x - 2 = 0$
$(x+2)(x-1) = 0$
$x = -2$, $x = 1$

12. If $f(x) = \frac{x^3}{6} - 2x$, then $f(x)$ has a minimum value at $x =$
a. 2
b. 0
c. −2
d. $\frac{8}{3}$

$f'(x) = \frac{3x^2}{6} - 2 = 0$
$\Rightarrow \frac{\cancel{3}x^2}{\cancel{6}_2} = 2$
$\Rightarrow x^2 = 4$
$\Rightarrow x = \pm 2$
$f''(x) = \frac{6x}{6} = x$
$f''(2) = 2 \Rightarrow \min$
$f''(-2) = -2 \Rightarrow \max$

13. If $f(x) = 4^{5x+1}$, then f′(0)=
a. 20 ln 5
b. 4 ln 5
c. 20 ln 4
d. None of the above

$f'(x) = 4^{5x+1}(\ln 4)\, 5$
$= 4^1 (\ln 4)\, 5$
$= 20 \ln 4$

14. The time required for an investment to be doubled at a rate of 20 % compounded continuously is

a. (0.1)(ln 2)years
b. (10)(ln 2)years
c. (5)(ln 2)years
d. None of the above

$P = P_0 e^{rt}$
$2P_0 = P_0 e^{rt}$
$2 = e^{0.2t}$
$\ln 2 = \ln e^{0.2t} \Rightarrow \ln 2 = 0.2t$
$\Rightarrow t = \frac{\ln 2}{0.2} = 5(\ln 2)$ years
$\frac{1}{0.2} = 5$

15. If $f(x,y) = e^{2x+3y}$ then $f_{xy} =$

a. $4e^{2x+3y}$
b. $9e^{2x+3y}$
c. $6e^{2x+3y}$
d. e^{2x+3y}

$f_x = \frac{\partial f}{\partial x} = 2e^{2x+3y}$
المشتقة بالنسبة x

$f_{xy} = \frac{\partial f}{\partial x \partial y}$
(اشتق بالنسبة) y
$= 2e^{2x+3y} 3$
$= 6 e^{2x+3y}$

16. If $f(x,y) = (3x + 2y)^3$ then $f_x(1,0) =$

a. 81
b. 54
c. 24
d. None of the above

17. If $f(x,y) = x^2y + \sqrt{xy}$ then $f_x(2,2) =$

a. $\frac{17}{2}$
b. $2 - \sqrt{2}$
c. $\frac{15}{2}$
d. $4 - \sqrt{2}$

16) $f_{(x,y)} = 3(3x+2y)^2 \, 3$
$= 9(3x+2y)^2 = 9(3)^2 = 81$

17) $f(x,y) = \frac{\partial f}{\partial x} = 2xy + \frac{y}{2\sqrt{xy}} = 2(2)(2) + \frac{\cancel{2}}{\cancel{2}\sqrt{4}}$
$= 8 + \frac{1}{2} = 8.5 = \frac{17}{2}$

هذا أوان الشد فاشتدي !!

Question # 2 (24%)

a If a bank pays 12% compounded quarterly, how much should be deposited now to have $6000 in five years.

$$P = P_0\left(1+\frac{j}{m}\right)^{mt}$$

$$6000 = P_0\left(1+\frac{0.12}{4}\right)^{(4)(5)}$$

$$\Rightarrow 6000 = P_0(1.03)^{20} \Rightarrow P_0 = 3322.05$$

b. Find the maximum and the minimum of f(x) = $x^4 - 8x^2$

$16 - 32 = -16$

$f'(x) = 4x^3 - 16x = 0$

$4x(x^2-4) = 0 \Rightarrow x = 0$ or $x = \pm 2$

$f''(x) = 12x^2 - 16$

$f''(0) = -16 \Rightarrow$ max at $(0,0)$

$f''(2) = f''(-2) = +32 \Rightarrow$ min at $(2,-16)$ $(2,16)$

(0,0) max

(2,-16) (2,16) min.

c. Formulate the following problem as a linear programming problem (DO NOT SOLVE):

A small accounting firm prepares tax returns for two types of customers: individuals and small businesses. Data is collected during an interview. A computer system is used to produce the tax return. It takes 3 hours to enter data into the computer for an individual tax return and 2.5 hours to enter data for a small business tax return. There is a maximum of 50 hours per week for data entry. It takes 30 minutes for the computer to process an individual tax return and 20 minutes to process a small business tax return. The computer is available for a maximum of 900 minutes per week. The accounting firm makes a profit of $200 on each individual tax return processed and a profit of $150 on each small business tax return processed. How many of each type of tax return should the firm schedule each week in order to maximize its profit?

LET The individuals be (I), Small business (S)

	I	S	
Hours	3	2.5	50
min.	30	20	900

$$3I + 2.5S \le 50$$

$$430I + 20S \le 900$$

$$\boxed{F = 200I + 150S}$$

$$I, S \ge 0$$

The monthly demand function for a product is given by $p + x = 140$, and its cost function is given by $C(x) = 1600 + 20x + x^2$

a. Find the equilibrium point

$D: P = 140 - x \Rightarrow R(x) = P \cdot x = 140x - x^2$

$C(x) = 1600 + 20x + x^2$

$\Rightarrow R(x) = C(x) \Rightarrow 140x - x^2 = 1600 + 20x + x^2$

$\Rightarrow 2x^2 + 120x + 1600 = 0 / 2$

$\Rightarrow x^2 + 60x + 800 = 0$

$(x - 20)(x - 40) = 0$

$x = 20, 40$ $\quad (20, 2400)$ $\quad (40, 4000)$

b. Find the maximum profit

$P(x) = R(x) - C(x) = 140x - x^2 - 1600 - 20x - x^2$

$= 120x - 2x^2 - 1600$

$P'(x) = 120 - 4x = 0 \Rightarrow 120 = 4x \Rightarrow x = 30$

$P''(x) = -4 \Rightarrow$ at $x = 30$ max

$P(30) = 120(30) - 2(30)^2 - 1600$

$= 200$ $\quad (30, 200)$ $\quad$ max Profit $= 200$

c. Find The minimum average cost per unit

$$AC(x) = \frac{1600}{x} + 20 + x$$

$$AC'(x) = \frac{-1600}{x^2} + 1 = 0 \Rightarrow \frac{+1600}{x^2} = +1$$

$\Rightarrow x^2 = 1600$

$AC'(x) = -1600x^{-2}$ $\quad x = \pm 40$

$AC''(x) = 3200x^{-3}$

$$= \frac{3200}{x^3} = \frac{3200}{64000} = 0.05 > 0 \Rightarrow \text{min at } x = 40$$

5

$$AC(40) = \frac{1600}{40} + 20 + 40$$

$AC(40) = 40 + 20 + 40 = \boxed{100}$

...estion # 4(15 %) The demand functions for two products are $p_1 = 12 - x$ and $p_2 = 20 - y$, where p_1 and p_2 are the respective prices (in thousands of dollars) for each product, and x and y are respective amounts (in thousands of units). Suppose the joint cost function is

$$C(x,y) = x^2 + 3y^2 + 2xy$$

Determine the prices and amounts that will maximize the profit. What is the maximum profit.

To Find Revenue: $P_1 x = 12x - x^2$, $P_2 y = 20y - y^2$

$\Rightarrow R(x,y) = 12x - x^2 + 20y - y^2$ ($P_1x + P_2y$ نجمع)

$C(x) = x^2 + 3y^2 + 2xy$

Profit $P(x,y) = R(x) - C(x)$

$= 12x + 20y - x^2 - y^2 - x^2 - 3y^2 - 3xy$

$= 12x + 20y - 2x^2 - 4y^2 - 2xy$

$\frac{\partial P}{\partial x} = 12 - 4x - 2y = 0 / 2$ --- (1)

$\frac{\partial P}{\partial y} = 20 - 8y - 2x = 0 / 2$ --- (2)

حل المعادلتين

$\Rightarrow -2x - y + 6 = 0 \Rightarrow -\cancel{2x} - y + 6 = 0$

$x - 2 \quad -x - 4y + 10 = 0 \Rightarrow \cancel{2x} + 8y - 20 = 0$

$7y - 14 = 0$

نعوض في أي معادلة $\Rightarrow \boxed{y = 2}$

$\Rightarrow -2x = y - 6 \Rightarrow +2x = +4 \Rightarrow \boxed{x = 2}$

To Determine Max. $\Rightarrow D = \frac{\partial^2 P}{\partial x^2} \frac{\partial^2 P}{\partial y^2} - \left(\frac{\partial^2 P}{\partial x \partial y}\right)^2$

$\frac{\partial^2 P}{\partial x^2} = -4$, $\frac{\partial^2 P}{\partial y^2} = -8$

$\frac{\partial^2 P}{\partial x \partial y} = -2$

6

$\Rightarrow D = (-4)(-8) - (-2)^2 = 32 - 4 = \boxed{28}$

$D > 0$, $\frac{\partial^2 P}{\partial x^2} = -4 < 0 \Rightarrow$ max at $x = 2$, $y = 2$

To Find the Prices $\Rightarrow P_1 = 12 - 2 = \boxed{10} \rightarrow$ for product (1)

$P_2 = 20 - 2 = \boxed{18} \rightarrow$ " " (2)

$P(x,y) = 12(2) + 20(2) - 2(2)^2 - 4(2)^2 - 2(2)(2) = \boxed{28} \rightarrow$ max profit